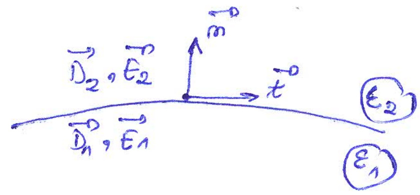
Rappel de Cours

1) • Composante normale de $\vec{D}$:

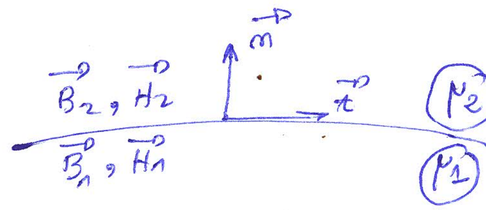


$$(\vec{D_2} - \vec{D_1}) \cdot \vec{n} = \sigma_\ell \xrightarrow[\sigma_\ell = 0]{\text{or}} \boxed{D_{n1} = D_{n2}}$$

•• Composante tangentielle de $\vec{D}$:

on a $(\vec{E_2} - \vec{E_1}) \wedge \vec{n} = 0 \Longrightarrow E_{t1} = E_{t2}$ D'où $\boxed{\dfrac{D_{t1}}{\varepsilon_1} = \dfrac{D_{t2}}{\varepsilon_2}}$

2) • Composante normale de $\vec{B}$:



$$(\vec{B_2} - \vec{B_1}) \cdot \vec{n} = 0 \Longrightarrow \boxed{B_{n1} = B_{n2}}$$

•• Composante tangentielle de $\vec{B}$:

on a $\vec{H}_{t2} - \vec{H}_{t1} = \vec{j}^{s}_{\ell} \wedge \vec{n} \xrightarrow[\vec{j}^{s}_{\ell} = \vec{0}]{\text{or}} H_{t2} = H_{t1}$

D'où $\boxed{\dfrac{B_{t1}}{\mu_1} = \dfrac{B_{t2}}{\mu_2}}$

I - Lame diélectrique l.h.i

1) Figure 1-a $\vec{E_0} = E_0 \vec{e_x}$ et $\vec{D_0} = \varepsilon_0 \vec{E_0}$

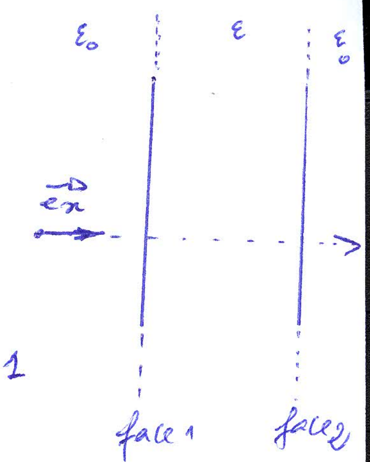
a) • $\vec{D} = D_n \vec{e_x} + D_t \vec{e_y}$

$$\begin{cases} D_t = \dfrac{\varepsilon}{\varepsilon_0} D_{t_0} = 0 \\ D_n = D_0 = \varepsilon_0 E_0 \end{cases} \Longrightarrow \boxed{\vec{D} = \varepsilon_0 \vec{E_0}}$$

• $\vec{E} = \dfrac{\vec{D}}{\varepsilon} \Longrightarrow \boxed{\vec{E} = \dfrac{\varepsilon_0}{\varepsilon} \vec{E_0}}$

• $\vec{E_p} = \vec{E} - \vec{E_0} \Longrightarrow \boxed{\vec{E_p} = \dfrac{(\varepsilon_0 - \varepsilon)}{\varepsilon} \vec{E_0}}$

b) • $\vec{P} = (\varepsilon - \varepsilon_0)\vec{E} \Rightarrow \boxed{\vec{P} = \frac{\varepsilon_0(\varepsilon - \varepsilon_0)}{\varepsilon}\vec{E_0}}$



• $\boxed{\rho_p = 0}$ car $\vec{P}$ uniforme.

• $\sigma_{p\,face1} = \vec{P}\cdot(-\vec{e_x}) \Rightarrow \boxed{\sigma_{p\,face1} = \frac{\varepsilon_0(\varepsilon_0 - \varepsilon)}{\varepsilon}E_0}$

• $\sigma_{p\,face2} = \vec{P}\cdot(\vec{e_x}) \Rightarrow \boxed{\sigma_{p\,face2} = \frac{\varepsilon_0(\varepsilon - \varepsilon_0)}{\varepsilon}E_0} = -\sigma_{p\,face1}$

Figure 1-b $\quad \vec{E_0} = E_0\vec{e_y}$

a)

• $\begin{cases} D_n = D_{n0} = 0 \\ D_t = \frac{\varepsilon}{\varepsilon_0}D_{t0} = \varepsilon E_0 \end{cases} \Rightarrow \boxed{\vec{D} = \varepsilon\vec{E_0}}$

• $\cancel{\vec{P} = (\varepsilon - \varepsilon_0)\vec{E}}$

• $\vec{E} = \frac{\vec{D}}{\varepsilon} \Rightarrow \boxed{\vec{E} = \vec{E_0}}$ $\qquad$ • $\boxed{\vec{E_p} = \vec{E} - \vec{E_0} = \vec{0}}$

b) • $\boxed{\vec{P} = (\varepsilon - \varepsilon_0)\vec{E_0}}$ $\qquad$ $\boxed{\bullet\ \rho_p = 0}$ $\qquad$ • $\boxed{\sigma_{p\,face_1} = \sigma_{p\,face2} = 0}$

Figure 1-c $\quad \vec{E_0} = E_0\cos\alpha\,\vec{e_x} + E_0\sin\alpha\,\vec{e_y} \qquad \vec{D_0} = \varepsilon_0\vec{E_0}$

a)

• $\begin{cases} D_n = D_{n0} = \varepsilon_0 E_0\cos\alpha \\ D_t = \frac{\varepsilon}{\varepsilon_0}D_{t0} = \varepsilon E_0\sin\alpha \end{cases} \Rightarrow \boxed{\vec{D} = E_0\left[\varepsilon_0\cos\alpha\,\vec{e_x} + \varepsilon\sin\alpha\,\vec{e_y}\right]}$

• $\begin{cases} E_t = E_{t0} = E_0\sin\alpha \\ E_n = \frac{\varepsilon_0}{\varepsilon}E_{n0} = \frac{\varepsilon_0}{\varepsilon}E_0\cos\alpha \end{cases} \Rightarrow \boxed{\vec{E} = E_0\left[\frac{\varepsilon_0}{\varepsilon}\cos\alpha\,\vec{e_x} + \sin\alpha\,\vec{e_y}\right]}$

• $\vec{E_p} = \vec{E} - \vec{E_0} \Rightarrow \boxed{\vec{E_p} = \frac{\varepsilon_0 - \varepsilon}{\varepsilon}E_0\cos\alpha\,\vec{e_x}}$

b)

• $\boxed{\vec{P} = (\varepsilon - \varepsilon_0)E_0\left[\frac{\varepsilon_0}{\varepsilon}\cos\alpha\,\vec{e_x} + \sin\alpha\,\vec{e_y}\right)}$ $\qquad$ • $\boxed{\rho_p = 0}$

• $\boxed{\sigma_{p\,face1} = -\frac{\varepsilon_0(\varepsilon - \varepsilon_0)}{\varepsilon}\cos\alpha\,E_0}$ $\qquad$ $\boxed{\sigma_{p\,face2} = -\sigma_{p\,face1}}$

2) • $\operatorname{tg}\beta = \dfrac{P_y}{P_x} = \dfrac{\sin\alpha}{\frac{\varepsilon_0}{\varepsilon}\cos\alpha} \Rightarrow \boxed{\operatorname{tg}\beta = \dfrac{\varepsilon}{\varepsilon_0}\operatorname{tg}\alpha}$

• on a $\varepsilon > \varepsilon_0 \Rightarrow \operatorname{tg}\beta > \operatorname{tg}\alpha$ d'où $\beta > \alpha$

3) $\begin{cases} \bullet \text{ pas de charge de polarisation} \\ \bullet\ \vec{E}_p = \vec{0} \end{cases}$

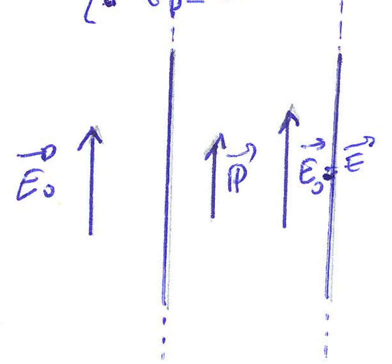
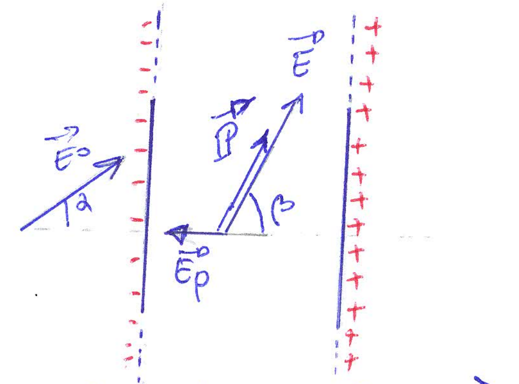


fig. 1. b



σ_p face 1 < 0 $\quad$ σ_p face 2 > 0

fig. 1. c

II - Lame magnétique l.h.i (7.5 pts)

1) a) • $\vec{B} = B_x\,\vec{e}_x + B_y\,\vec{e}_y$

• $\begin{cases} B_x = B_{0x} \\ \dfrac{B_y}{\mu} = \dfrac{B_{0y}}{\mu_0} \end{cases} \Rightarrow \boxed{\vec{B} = B_{0x}\,\vec{e}_x + \dfrac{\mu_0(1+\chi_m)}{\mu_0} B_{0y}\,\vec{e}_y}$ 1

•• $\vec{H} = H_x\,\vec{e}_x + H_y\,\vec{e}_y \qquad \vec{H} = \dfrac{\vec{B}}{\mu}$

• $\begin{cases} H_x = \dfrac{B_{0x}}{\mu} \\ H_y = \dfrac{B_{0y}}{\mu_0} \end{cases} \Rightarrow \boxed{\vec{H} = \dfrac{B_{0x}}{\mu}\,\vec{e}_x + \dfrac{B_{0y}}{\mu_0}\,\vec{e}_y}$ 0.5

b) $\vec{M} = \chi_m \vec{H} \Rightarrow \boxed{\vec{M} = \dfrac{\chi_m}{\mu_0(1+\chi_m)} B_{0x}\,\vec{e}_x + \dfrac{\chi_m}{\mu_0} B_{0y}\,\vec{e}_y}$ 0.5

c) • $\operatorname{tg}\beta = (1+\chi_m)\dfrac{B_{0y}}{B_{0x}} \Rightarrow \boxed{\operatorname{tg}\beta = (1+\chi_m)\operatorname{tg}\alpha}$ 0.5

$\begin{cases} \bullet \text{ si lame diamagn. } \chi_m < 0 \Rightarrow \operatorname{tg}\beta < \operatorname{tg}\alpha \Rightarrow \beta < \alpha \\ \bullet \text{ si lame paramag. } \chi_m > 0 \Rightarrow \operatorname{tg}\beta > \operatorname{tg}\alpha \Rightarrow \beta > \alpha \end{cases}$ 0.5

Rq. $|\chi_m| \ll 1 \Rightarrow \operatorname{tg}\beta \simeq \operatorname{tg}\alpha \Rightarrow \beta \simeq \alpha$ pour les milieux para. et dia.

2) a) Cas $\alpha = 0$ $\quad \vec{B_0} = B_{0x}\,\vec{e_x}$ (chp normal à la lame)

0.25 • $\vec{B} = \vec{B_0} = B_{0x}\,\vec{e_x}$ • $\vec{H} = \frac{\vec{B_0}}{\mu} = \frac{B_{0x}}{\mu}\,\vec{e_x}$ 0.25

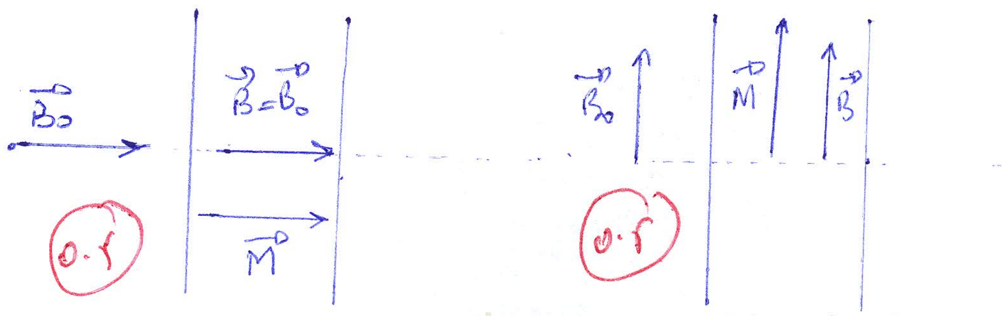
0.25 • $\vec{M} = \chi_m\,\vec{H} = \frac{\chi_m}{\mu_0(1+\chi_m)}\,B_{0x}\,\vec{e_x}$

Cas $\alpha = \pi/2$ $\quad \vec{B_0} = B_{0y}\,\vec{e_y}$ (chp parallèle à la lame)

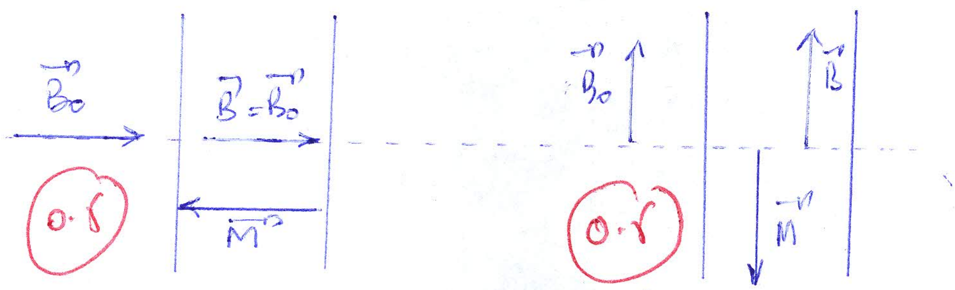
0.25 • $\vec{B} = (1+\chi_m)\,B_{0y}\,\vec{e_y}$ • $\vec{H} = \frac{B_{0y}}{\mu_0}\,\vec{e_y}$ 0.25

• $\vec{M} = \frac{\chi_m}{\mu_0}\,B_{0y}\,\vec{e_y}$ 0.25

b) Cas $\alpha = 0$ $\qquad$ Cas $\alpha = \pi/2$



(i) milieu paramagnétique ($\chi_m > 0$)



(ii) milieu diamagnétique ($\chi_m < 0$)

3) • densité de courant volumique d'aimantation : $\vec{j}^{\,m}_{v} = \vec{rot}\,\vec{M} = \vec{0}$ car $\vec{M}$ uniforme.

• densité " " surfacique : face 1 $\quad \vec{j}^{\,m}_{s_1} = \vec{M} \wedge (-\vec{e_x}) = M_y\,\vec{k}$

$\boxed{\vec{j}^{\,m}_{s_1} = \frac{\chi_m}{\mu_0}\,B_{0y}\,\vec{k}}$ 0.5

face 2 $\quad \boxed{\vec{j}^{\,m}_{s_2} = -\frac{\chi_m}{\mu_0}\,B_{0y}\,\vec{k}}$ 0.5