

Exercice N°1

$$s(t) = 50u(t) + 100tu(t) - 100(t - 0,5)u(t - 0,5) + 50u(t - 2) + 100(t - 2)u(t - 2) - 200(t - 2,5)u(t - 2,5) + 100(t - 3)u(t - 3) - 150u(t - 3) + 150 \sin(2\pi(t - 4))u(t - 4) + 150 \sin(2\pi(t - 4,5))u(t - 4,5)$$

$$S(p) = \frac{1}{p} + \frac{1}{p^2} - \frac{100e^{-0,5p}}{p^2} + \frac{50e^{-2p}}{p} + \frac{100e^{-2p}}{p^2} - \frac{200e^{-2,5p}}{p^2} + \frac{100e^{-2p}}{p^2} - \frac{150e^{-2p}}{p} + \frac{300\pi e^{-4p}}{p^2+4\pi^2} + \frac{300\pi e^{-4,5p}}{p^2+4\pi^2}$$

Exercice N°2

a) $\frac{dy(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = tu(t)$; avec : $y(0)=1$ et $\dot{y}(0) = 1$

$$\Rightarrow p^2 Y(p) - p - 1 + 5pY(p) - 5 + 4Y(p) = \frac{1}{p^2}$$

$$\Rightarrow Y(p) = \frac{p^3+6p^2+1}{p^2(p^2+5p+4)} = \frac{p^3+6p^2+1}{p^2(p+4)(p+1)} = \frac{a}{p^2} + \frac{b}{p+4} + \frac{c}{p+1}$$

On obtient : $a = \frac{1}{4}$; $b + c + d = 1$; $c = -\frac{11}{16}$; $d = 2 \Rightarrow b = -\frac{5}{16}$

$$\Rightarrow s(t) = -\frac{5}{16} + \frac{t}{4} - \frac{11e^{-4t}}{16} + 2e^{-2t}$$

b) $\frac{dy(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2(\cos 4t)u(t)$; avec : $y(0)=1$ et $\dot{y}(0) = 1$

$$\Rightarrow p^2 Y(p) - p - 1 + 4pY(p) - 4 + 3Y(p) = \frac{2p}{p^2+16}$$

$$\Rightarrow Y(p) = \frac{(p+5)(p^2+16)+2p}{(p^2+16)(p^2+4p+3)} = \frac{(p+5)(p^2+16)+2p}{(p^2+16)(p+3)(p+1)} = \frac{ap+b}{p^2+16} + \frac{c}{p+3} + \frac{d}{p+1}$$

On obtient : $c = -\frac{22}{25}$; $d = \frac{33}{17}$

Puis : $\frac{(p+5)(p^2+16)+2p}{(p^2+16)(p+3)(p+1)} = \frac{ap+b}{p^2+16} - \frac{22}{25(p+3)} + \frac{33}{17(p+1)}$

Pour $p = 0$ dans l'expression précédente on obtient : $\frac{5}{3} = \frac{b}{16} - \frac{22}{75} + \frac{33}{17} \Rightarrow b = -\frac{128}{17 \times 25}$

Pour $p = 1$ dans l'expression précédente on obtient :

$$\frac{13}{17} = \frac{a+b}{17} - \frac{11}{50} + \frac{33}{34} \Rightarrow a = 13 - b + \frac{11 \cdot 17}{50} - \frac{33}{2} = \frac{46}{85}$$

$$\Rightarrow Y(p) = \frac{46/85 p + 128/425}{p^2+16} - \frac{22}{25(p+3)} + \frac{33}{17(p+1)}$$

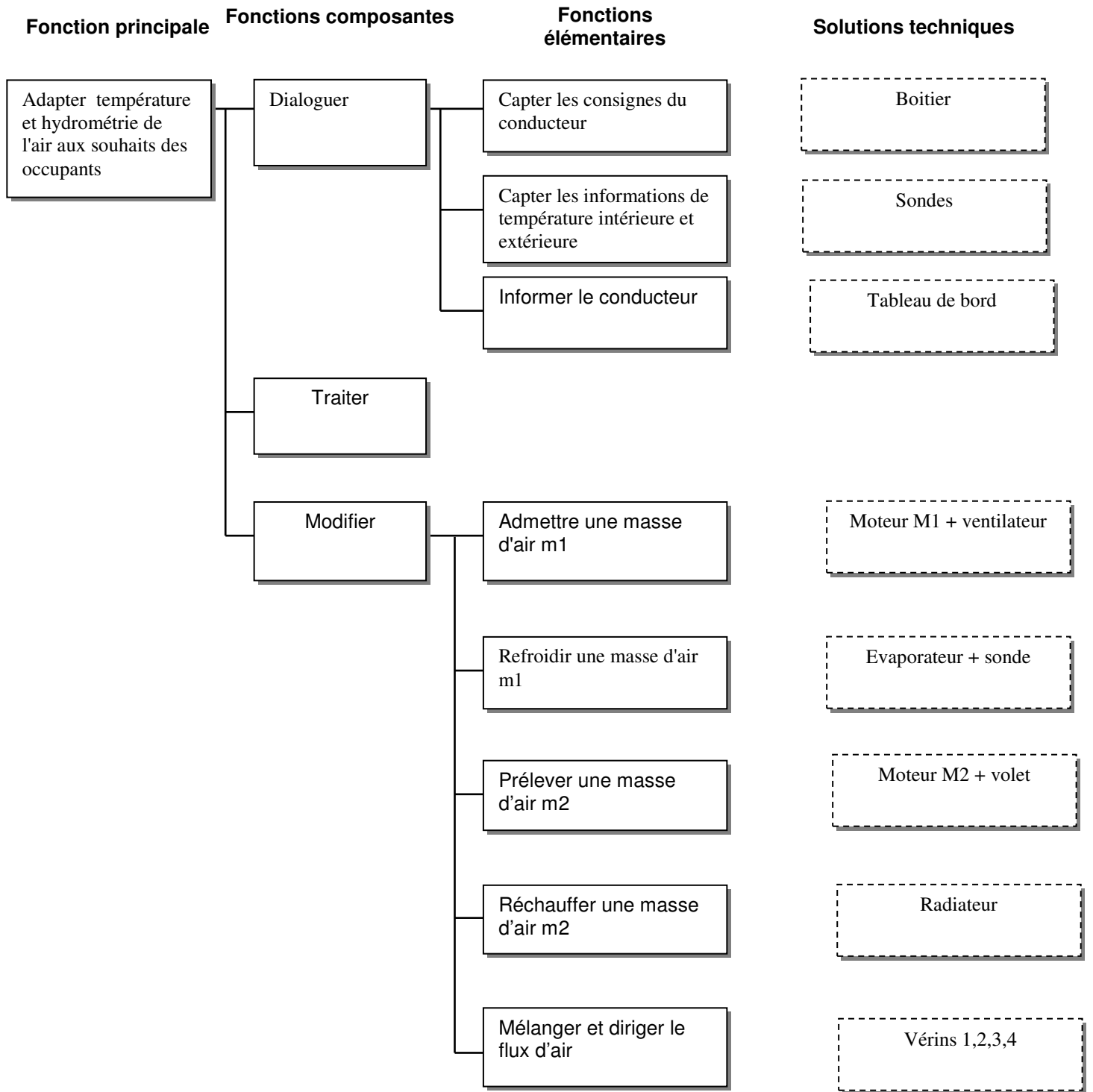
$$\Rightarrow Y(p) = \frac{46/85 p + 128/425}{p^2+16} - \frac{22}{25(p+3)} + \frac{33}{17(p+1)}$$

$$\Rightarrow y(t) = \frac{46}{85} \cos 4t + \frac{22}{100} \sin 4t - \frac{22}{25} e^{-3t} + \frac{33}{17} e^{-t}$$

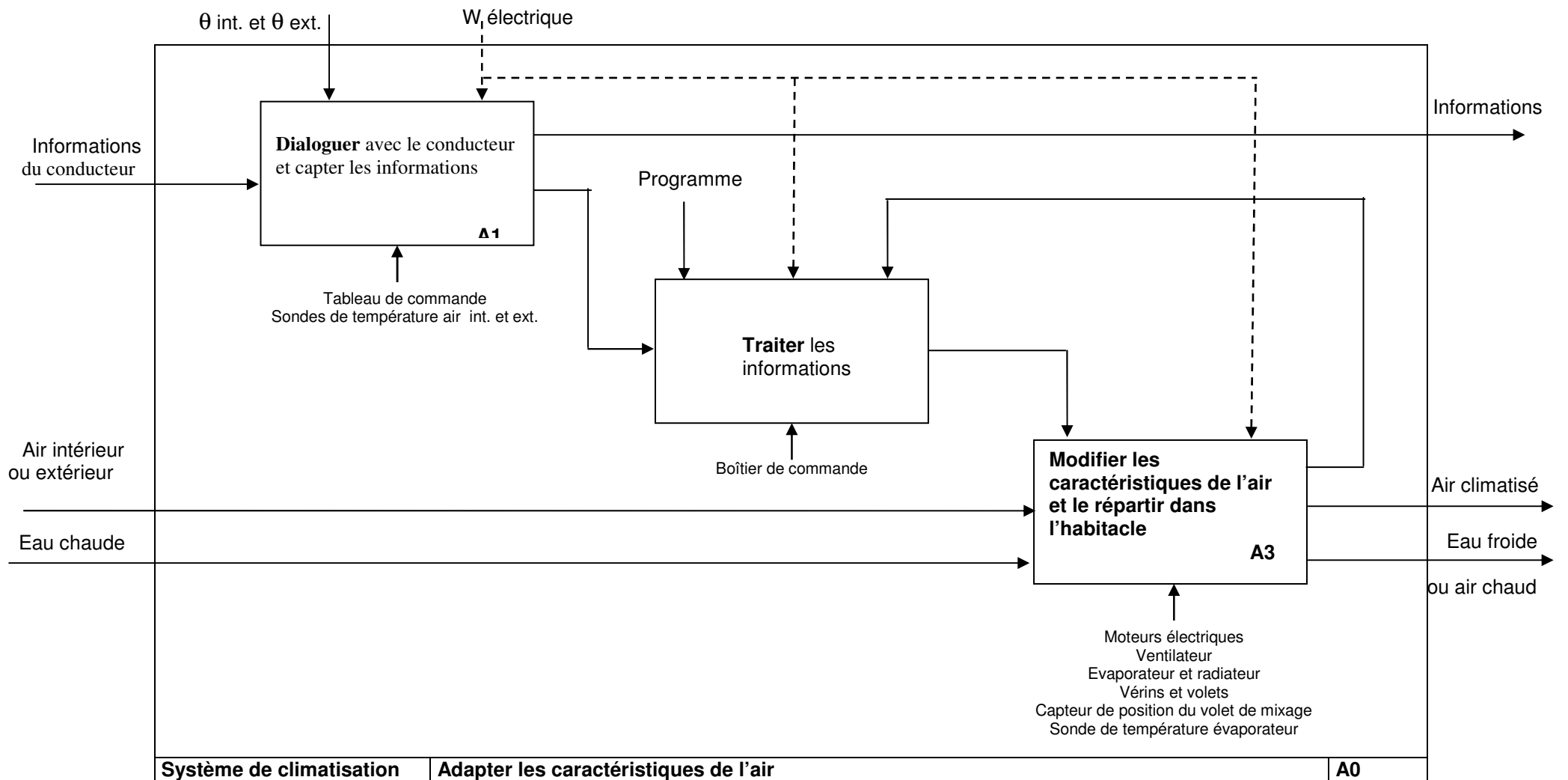
Exercice N°3

$$\lim_{t \rightarrow 0} f(t) = \lim_{p \rightarrow \infty} pF(p) = \lim_{p \rightarrow \infty} \frac{2p^3 + 2p^2 + 3p + 7}{3(2p + 0,5)(p^2 + 4)} = \frac{1}{3}$$

$$\lim_{t \rightarrow \infty} f(t) = \lim_{p \rightarrow 0} pF(p) = \lim_{p \rightarrow 0} \frac{2p^3 + 2p^2 + 3p + 7}{3(2p + 0,5)(p^2 + 4)} = \frac{7}{6}$$



DOCUMENT REPONSE 2



DOCUMENT REPOSE 3

